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FERRITE PHASE SHIFTER FINITE ELEMENT ANALYSIS INCLUDING LOSSES

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ABSTRACT

Transmission lines of arbitrary cross-section loaded by dielectric and gyromagnetic lossy media of arbitrary shape are analyzed. A finite element method using a new transerve fields formulation is used, which leads to an eigenvalue problem. This direct method allows to take into account complex frequency dependent physical parameters. Numerical examples are carried out on a ferrite phase shifter . Attenuation of the guided wave is evaluated.

I - INTRODUCTION

This issue presents a studie [1-6] based on a finite element formulation of 2D transmission line problem. A novel method is proposed, using a new "transverse fields" basis. At this time, direct calculation of propagation parameters for a 2D transmission line of arbitrary cross-section, loaded by ferrites and dielectrics with arbitrary sections, was not resolved in its whole generality : some papers give approaches to this problem but always with important simplifications. At present state of this work, following hypothesis are made : metal walls are assumed to have perfect conductivity ; permittivity is supposed to be constant in each material, but may vary from material to material and can be very high ; there are no charges and no currents inside the wave-guide. Main interests of this formulation are :
- to be direct, i.e. that frequency is considered as a data, and the complex propagation constant Γ becomes the unknown parameter.
- to give a first degree equation in Γ.

So this formulation allows both direct introduction of physical parameters which are most often frequency dependent, and also to take into account losses of materials and their variations throughout samples.

II - VARIATIONAL FORMULATION

Basic equations describing an electromagnetic wave propagating in z direction are normalized using complex vectorial variables $\vec{E}$.exp $(j\omega t-\Gamma z)$ and $\vec{G}$. exp $(j\omega t-\Gamma z)$, with $\vec{G} = -j.c.\mu o.\vec{H}$, where c is the light velocity in free space and μo is the permeability of free space.
Thus, Maxwell equations become

$$\begin{cases} \frac{c}{\omega}\vec{\nabla}\times\vec{E} = \bar{\bar{\mu}}_r\,\vec{G} \\ \frac{c}{\omega}\vec{\nabla}\times\vec{G} = \epsilon_r\,\vec{E} \end{cases} \qquad \begin{cases} \vec{\nabla}.\vec{E} = 0 \\ \vec{\nabla}.(\bar{\bar{\mu}}_r\,\vec{G}) = 0 \end{cases}$$

where ω is the angular frequency, εr is the relative scalar permittivity and $\bar{\bar{\mu}}r$ is the relative permeability tensor of the medium.
The mathematical development is described in [7] and leads to the following system :

$$\begin{cases} \Gamma\vec{E}_t = -\vec{\nabla}_t E_z + \frac{\omega}{c}\mu_{zz}\,\vec{z}\times\vec{G}_t & (1) \\ \Gamma\vec{G}_t = -\vec{\nabla}_t G_z + \frac{\omega}{c}\epsilon_r\,\vec{z}\times\vec{E}_t & (2) \end{cases}$$

$$\vec{\nabla}_t = \begin{pmatrix}\partial/\partial x \\ \partial/\partial y \\ 0\end{pmatrix},\ \vec{z} = \begin{pmatrix}0\\0\\1\end{pmatrix},\ \bar{\bar{\mu}}_r = \begin{pmatrix}\bar{\bar{\mu}}_{tt} & \bar{\bar{\mu}}_{tz} \\ \bar{\bar{\mu}}_{zt} & \mu_{zz}\end{pmatrix}$$

$$E_z = \frac{c}{\omega\epsilon_r}\left(\vec{\nabla}_t\times\vec{G}_t\right).\vec{z} \qquad (3)$$

$$G_z = \left[\frac{c}{\omega\mu_{zz}}\left(\vec{\nabla}_t\times\vec{E}_t\right) - \frac{\bar{\bar{\mu}}_{zt}}{\mu_{zz}}\vec{G}_t\right].\vec{z} \qquad (4)$$

$$\mu_{zz}\vec{G}_t = \frac{c}{\omega}\frac{\bar{\bar{\mu}}_{tz}}{\mu_{zz}}\left(\vec{\nabla}_t\times\vec{E}_t\right) + \left(\bar{\bar{\mu}}_{tt} - \frac{\bar{\bar{\mu}}_{tz}\bar{\bar{\mu}}_{zt}}{\mu_{zz}}\right)\vec{G}_t \qquad (5)$$

where the subscripts tt, tz, zt, and zz refer to the 2x2,2x1,1x2,1x1 submatrices.
After multiplying (1) and (2) with trial functions $\vec{U}t$, $\vec{V}t$ and integrating by parts the expressions over the waveguide section, one can suppress secondary derivatives :

$$\begin{aligned}\Gamma\iint_s \vec{U}_t\otimes\vec{E}_t\,ds = \ & \frac{\omega}{c}\iint_s \mu_{zz}\,\vec{U}_t\otimes(\vec{z}\wedge\vec{G}_t)\,ds \\ & + \iint_s E_z\,\vec{\nabla}_t\otimes\vec{U}_t\,ds \\ & + \int_c E_z\,\vec{U}_t\otimes(\vec{z}\wedge d\vec{l}) \quad (6)\end{aligned}$$

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$$\Gamma \iint_S \vec{V_t} \otimes \vec{G_t}\, ds = \frac{\omega}{c} \iint_S \epsilon_r \vec{V_t} \otimes (\vec{z} \wedge \vec{E_t})\, ds + \iint_S G_z \vec{V_t} \otimes \vec{V_t}\, ds + \int_C G_z \vec{V_t} \otimes (\vec{z} \wedge d\vec{U}) \quad (7)$$

$$\vec{A_t} \otimes \vec{B_t} = \begin{pmatrix} A_x.B_x \\ A_y.B_y \\ 0 \end{pmatrix} \qquad d\vec{U} = \begin{pmatrix} dx \\ dy \\ 0 \end{pmatrix}$$

Longitudinal fields Ez and Gz are expressed as functions of transverse fields with expressions (3) and (4).
This formulation allows to take into account dielectric and magnetic losses. No restriction is made on permeability. Actual calculations are made with a scalar permittivity which is supposed to be constant in each material but formulation is compatible with a tensorial and locally variable permittivity.

III - FINITE-ELEMENT DISCRETIZATION AND BOUNDARY CONDITIONS

Trial functions $\vec{Ut}$ and $\vec{Vt}$ are chosen continuous on the integration region, with the same external boundary conditions as the physical variables $\vec{Et}$ and $\vec{Gt}$. These last ones are expressed as function ot $\vec{Ut}$ and $\vec{Vt}$. However, $\vec{Et}$ and $\vec{Gt}$ are discontinuous on the boundaries between two different media. These discontinuities can be treated by the use of a matrix transformation [8] on the selected side of the boundary. This is possible because the boundary steps of transverse fields are explicitely known.
According to the finite element method, equations (6) and (7), with Ez, Gz and $\vec{Gt}$ given by (3), (4) and (5), are discretized into a finite summation, so that unknown variables become the discrete values of $\vec{Et}$ and $\vec{Gt}$ at each nodes of the meshing (cf. figures 1 and 3) :

$$\begin{pmatrix} \vec{E_t} \\ \vec{G_t} \end{pmatrix}_i = [P]_{ij} \{X_i\} \;, \quad \begin{pmatrix} \vec{E_t} \\ \vec{G_t} \end{pmatrix}_j = \{X_j\}$$

If Et and Gt are continuous at the boundary between materials i and j, then [Pij] is the identity matrix.
Above equations are reduced to a linear system :

$$[A].\{X_i\} = \Gamma.\,[B].\,\{X_i\}$$

Waveguide boundaries are introduced by DIRICHLET conditions.
Triangular finite elements of the second order were employed for discretization of the field region in order to calculate the longitudinal components Ez and Gz. Indeed, these components are necessary to evaluate losses and power effects.

IV - ACTUAL NUMERICAL RESULTS FOR LOSSLESS CASES

Dielectric slab loaded waveguide case :
In order to verify the software, it was first tested with a simple dielectric loaded waveguide. Results where compared with analytical calculated values, and precision of the fundamental mode was always better than 0.03 %.
Ferrite slab loaded waveguide case :
A similar ferrite slab loaded waveguide structure (dimensions : 8.64x4.32 mm2) is studied. This structure, shown on figure (1), is composed of air (material 1) and of a ferrite slab (material 2) characterized by its relative permittivity $\varepsilon r=2$, and its tensorial permeability :

$$\overline{\overline{\mu r}} = \begin{bmatrix} 1 & 0 & 0.1j \\ 0 & 1 & 0 \\ -0.1j & 0 & 1 \end{bmatrix}$$

Main results on TE10 modes and other bounded modes are presented on figure (2) and compared with theoretical computed values. The meshing here used has got 114 nodes. Calculations are made on MICROVAX II.
Again, our finite element calculations are in in good agreement with the exact solutions for the fundamental mode. At high frequencies, precision on higher order modes is degraded, due to an insufficient number of elements in the meshing.
Actually, all spurious can be eliminated with physical criteria on real and imaginary parts of the propagation constants.
The same exemple has been carried out at 25 GHz with a "166 nodes" mesh and propagation constant was more precise : $\delta\beta$ = 17.413 rad/m, precision = 0,8 %.

V - ACTUAL RESULTS WITH LOSSY MATERIALS

Influence of dielectric and magnetic losses on the complex propagation constant Γ has been evaluated for a ferrite loaded waveguide at 25 GHz. The mesh is presented on figure (3). The ferrite slab (material 2) is magnetized in order to get a relative permeability tensor

$$\overline{\overline{\mu r}} = \begin{bmatrix} \mu & 0 & jk \\ 0 & 1 & 0 \\ -jk & 0 & \mu \end{bmatrix}$$

with $\mu = 0{,}8-j\mu''$ and $k = 0{,}4-jk''$
The relative permittivity of the ferrite has a realistic value of : $\varepsilon_r = 10 - j.\varepsilon''$.
Comparison of finite element loss contant and corresponding semi-analytical calculated value gives good agreement either for dielectric or magnetic losses as shown on figure 4.

VI - DESIGN OF FERRITE PHASE SHIFTERS

Precise calculations of ferrite phase shifters need a good knowledge of both local static internal fields and magnetization inside the ferrite.

So, a finite element sub-process is necessary to evaluate these vectorial quantities. This sub-process must use the same meshing as for the propagation calculation. Then, a routine calculates the locally variable permeability tensor, which is necessary in order to get precise results.
Arcing phenomena and non linear effects inside the ferrite are related to electromagnetic local fields. Local heating inside dielectric and magnetic materials can also be deduced. As electromagnetic local fields and local losses are calculated, this formulation allows to evaluate mean and peak power capability of devices. A graphic post processor will give useful informations on engineering problems.

VII - CONCLUSION

This electromagnetic fields 2D formulation allows, in microwave propagation studies by finite elements methods, to introduce at each point of a transmission line section any frequency dependant parameter as tensor $\overline{\overline{\mu r}}$ of magnetically polarized ferrite.
Applied on microwave phase-shifter structures fully analytically computable, first results show very precise values of propagation constants both for dielectric or ferrite loaded cases. Evaluation of more real phaseshifters are on the way, but they need to evaluate local magnetization and local magnetostatic field. Results will be given at the conference with presentation of maps of local fields and local losses.
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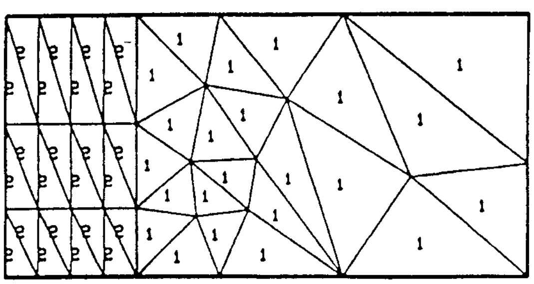

FIGURE - 1 -

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TE10 MODES : PROPAGATION CONSTANT VERSUS FREQUENCY				
FREQ. (GHz)	ß THEORETICAL VALUES (rad/m)	ß CALCULATED VALUES (rad/m)	PREC. (%)	HIGHER ORDER CALCULATED MODES (% Precision on ß)
25	ß+ = 427.157 ß_ = -409.882 δß = 17.275	ß+ = 427.398 ß_ = -409.594 δß = 17.804	0.06 0.07 3.	NO HIGHER MODE
30	ß+ = 574.410 ß_ = -553.699 δß = 20.711	ß+ = 574.524 ß_ = -553.570 δß = 20.954	0.02 0.02 1.2	LSM11 (*)
35	ß+ = 716.721 ß_ = -691.410 δß = 25.311	ß+ = 716.550 ß_ = -691.455 δß = 25.095	0.02 0.007 0.9	LSM11 (*) LSE20 (0.43%)
40	ß+ = 860.584 ß_ = -830.029 δß = 30.555	ß+ = 860.003 ß_ = -830.394 δß = 29.609	0.07 0.04 3.1	LSM11 (*) LSE20 (0.23%) LSM21 (*) LSE10 (*)
45	ß+ = 1008.504 ß_ = -973.404 δß = 35.1	ß+ = 1007.283 ß_ = -974.086 δß = 33.197	0.12 0.07 5.4	LSM11 (*) LSE11 (*) LSM21 (*) LSE20 (0.16%)

(*) Theoretical values are not available

FIGURE - 2 -

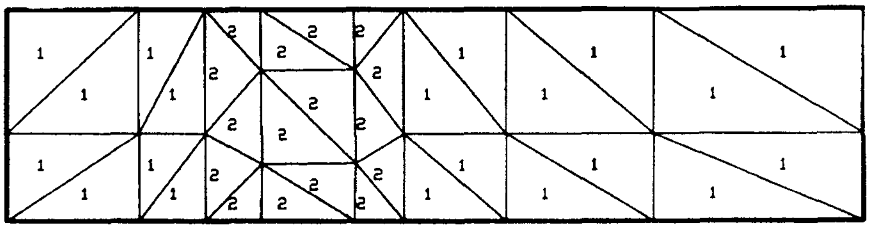

FIGURE - 3 -

LOSSES	SEMI-ANALYTICAL METHOD α (dB/m)	FINITE-ELEMENT METHOD α (dB/m)
ϵ''= 1.10-3 μ''= k''= 0	$\alpha-$ = 5.66 $\alpha+$ = 5.66	$\alpha-$ = 5.59 $\alpha+$ = 5.69
μ''= 8.10-4 ϵ''= k''= 0	$\alpha-$ = 5.31 $\alpha+$ = 5.36	$\alpha-$ = 5.25 $\alpha+$ = 5.38
k''= 4.10-4 ϵ''= μ''= 0	$\alpha-$ = 1.46 $\alpha+$ = 1.48	$\alpha-$ = 1.42 $\alpha+$ = 1.50

FIGURE - 4 -